3D-Rotation of any vector (x,y,z) around an axis of the direction(a,b,c) by an angel@

We reduce the vector of the axis-direction to the length 1:
$(1/ \sqrt{a^2+b^2+c^2}) * (a,b,c)=(A,B,C)$.
Reckon the following and you get the result of the rotation

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin@ * \begin{pmatrix} 0 & -C & B \\ C & 0 & -A \\ -B & A & 0 \end{pmatrix} + (1-\cos@) * \begin{pmatrix} 0 & -C & B \\ C & 0 & -A \\ -B & A & 0 \end{pmatrix}^2 \right] * \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(Notice, that the third matrix must be squared and then multiplied by cos@)
Imagine a plane, to which the axis is normal to and in which lies the tip of the arrow (that is the picture of the vector)In this plane you add an arrow from the tip in the direction of travel -that is the orientation of the rotation. And from this you add another one in this plane in the direction of 90 degrees to the left respective to the privious one.
The vector (x,y,z) and the result of the formula above are of same length. The angle between these two is not the angle of rotation - the tip of the arrow is rotated in the plane , which is perpendicular to the axis.

Extract axis and angle out of a rotation-matrix

A rotation-matrix D has the property: det D =1 and $D^T * D = E$, where D^T is the matrix transponed, that is you interchanged colums and rows and E is the unit-matrix.
A matrix can be split into a symmetrical ($a_{ik} = a_{ki}$) and an antisymmetrical ($a_{ik} = -a_{ki}$)

$$\begin{pmatrix} a & d & e \\ g & b & f \\ h & i & c \end{pmatrix} = 1/2 * \begin{pmatrix} 2a & d+g & e+h \\ d+g & 2b & f+i \\ e+h & f+i & 2c \end{pmatrix} + 1/2 * \begin{pmatrix} 0 & d-g & e-h \\ g-d & 0 & f-i \\ h-e & i-f & 0 \end{pmatrix}$$

The antisymmetrical part gives the direction af the axis: (i-f, e-h, g-d)*1/2.
The length of this is sin@.
The main diagonal of the matrix gives the "spur": a+b+c and this equals 1 + 2*cos@. From these you get @.
An extra-bonus: The affin mappings (if this is the rigt word), that is here the 3*3-matrices can be split in an symmetrical and an antisymmetrical part. The first you explore by means of main-axis transformation and the antisymmetrical ones - applied to a vector - correspond to the cross-product:

$$\begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (a, b, c) \times (x, y, z)$$

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